Section 3.4 Properties of Logarithms

Change-Base Formula:

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then $\log_a x$ can be converted to a different base as follows.

Base b

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Base 10

$$\log_a x = \frac{\log x}{\log a}$$

Base e

$$\log_a x = \frac{\ln x}{\ln a}$$

Ex 1: Evaluate each of the following, using the change-of-base formula with common logs. Approximate to four decimal places.

a. $\log_3 16 = \frac{\log(16)}{\log(3)} = 2.5237$

b. $\log_5 22 = \frac{\log(22)}{\log(5)} = 1.9206$

* log BASE button in MATH

Ex 2: Evaluate each of the following, using the change-of-base formula with natural logarithms. Approximate to four decimal places.

a. $\log_3 16 = \frac{\ln(16)}{\ln(3)} = 2.5237$

b. $\log_5 22 = \frac{\ln(22)}{\ln(5)} = 1.9206$

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, then the following properties are true.

	Logarithm with Base a	Natural Logarithm
1. Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln \frac{u}{v} = \ln u - \ln v$
3. Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

Ex 3: Write each logarithm in terms of ln2 and ln5. Expand

a. $\ln 10 = \ln(2 \cdot 5) = \ln 2 + \ln 5$

b. $\ln \frac{5}{32} = \ln \frac{5}{2^5} = \ln 5 - 5\ln 2$

Ex 4: Find the exact value of each expression without using a calculator.

a. $\log_7 \sqrt[5]{7}$

$\log_7 7^{1/5} = \boxed{\frac{1}{5}}$

$7^{?} = 7^{1/5}$

b. $\ln e^{12} + \ln e^{5}$

$\ln(e^{12} \cdot e^{5}) = \ln(e^{17}) = 17$

$(\log_e e^{17}) \rightarrow e^{?} = e^{17}$

Ex 5: Expand each logarithmic expression.

a. $\log(3x^2y)$

$\log 3 + \log x^2 + \log y$

$\boxed{\log 3 + 2\log x + \log y}$

b. $\ln \frac{\sqrt{4x+1}}{8} = \ln \frac{(4x+1)^{1/2}}{8}$

$\ln(4x+1)^{1/2} - \ln 8$

$\boxed{\frac{1}{2}\ln(4x+1) - \ln 8}$

Ex 6: Condense each logarithmic expression.

a. $\frac{1}{3}\log x + 5\log(x-3)$

$\log x^{1/3} \cdot (x-3)^5 = \boxed{\log \sqrt[3]{x}\,(x-3)^5}$

b. $4\ln(x-4) - 2\ln x$

$\boxed{\ln\left(\frac{(x-4)^4}{x^2}\right)}$

c. $\frac{1}{5}[\log_3 x + \log_3(x-2)]$

$\rightarrow \frac{1}{5}\log_3 x + \frac{1}{5}\log_3(x-2)$

$\log_3 x^{1/5} \cdot (x-2)^{1/5}$

$\boxed{\log_3 \sqrt[5]{x(x-2)}}$